## Background

Calculus II

This is a continuation of Calculus I (MAT1512). It deals with the mathematics of change.

Outcomes:

- Calculate and use the derivatives of a function to sketch a graph of the function

- First derivative. Determine the relationship between the rates of change of various quantities in the rates-of-change word problem.

- Solve maximum or minimum word problems using the theory of derivatives.

- Ability to use L’Hopital’s rule to determine limits of indeterminate forms.

- Calculation of the volumes of solids of revolution.

- An improper integral is tested for convergence or divergence and evaluated if convergent.

- Integration techniques to evaluate integrals.

- Taylor polynomial of any order at a given point.

A close up of a map

Description automatically generated

**Lesson 0**

Revision: Limits

Limits are the value a function approaches as the input “approaches” some value. They are used to define continuity, derivatives, and integrals.

We do not care about the output (of a function) at a certain point, but more what happens around the point.

Limits help solve the problem of indeterminate form

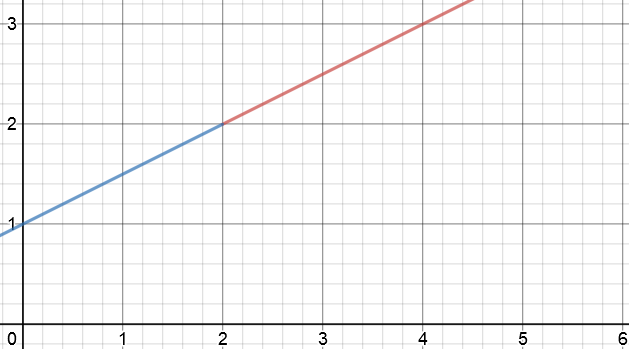
Calculating instantaneous velocity is an example of a limit

Example: The function and the limit differ

*LHS: The limit as x approaches 2 is 2*

*RHS: The limit as x approaches 2 is 2*

*The limit as x approaches 2 is 2*

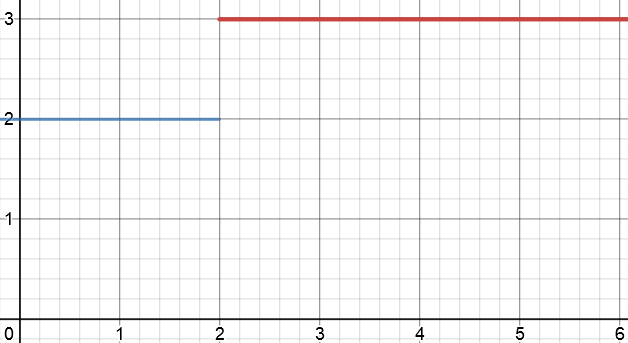


Example: The LHS and RHS limit differ

*LHS: The limit as x approaches 2 is 2*

*RHS: The limit as x approaches 2 is 2*

*The limit as x approaches 2 does not exist*



A close up of a device

Description automatically generatedExample: The limits at inifinity

*LHS: The limit as x approaches -infinity is -1*

*RHS: The limit as x approaches infinity is -1*

*Reciprocal graph. Asymptotes @ x=2, y=-1*

<https://www.youtube.com/watch?v=nJZm-zp639s>

**Common** **(PFGE)** Use these methods in order. If one fails, try the next

[1] Plug in values

*Always start by plugging in the x value*

[2] Algebra. Factorization

*Indeterminate form . Factorize and the plug-in x value*

If you use [2] and you get an answer over zero, then DNE (does not exist)

Other DNE examples

[3] Algebra. Get common Denominator

*Reciprocal Substitute*

[4] Expand Parentheses

*Expand then simplify*

**Uncommon (STA)**

[5] Square root in numerator (in rational expression)

*Multiply by conjugate (differentiation). Remember to change sign of 2nd term*

[6] Trig functions (indeterminate form)

*Special property: or*

*Special property: or*

*because*

[7] Absolute Value

*Piecewise definition of ABS function:*

*Find see if LHS limit = RHS limit*

<https://www.youtube.com/watch?v=nViVR1rImUE>

**Limits at infinity**

[8] Polynomial/Constant

*Lower degree terms (2x and 5) irrelavant matter here*

[9] Rational

*degree\_N < degree\_D*

*ratio of leading coefficients*

*degree\_N = degree\_D*

*degree\_N > degree\_D*

[10] Trig functions

*Special property: or*

*Special property: or*

*also 0*

[11] Exponential

*eval:*

*eval:*

*Sub*

*Sub*

[11] L’Hopital’s Rule

*Special property:*

Example: natural logs

**Continuity**

A function is continuous at a if

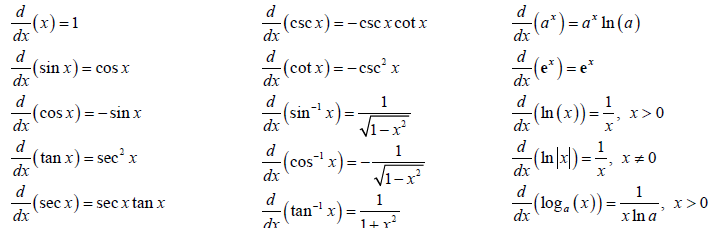
*the limit of the function at input a is defined*

**Lesson 0**

Revision: Derivatives

Derivatives are the slope of a function. It calculates the instantaneous rate of change at each point of the old function.

Common derivatives:



What is and what is the difference between and ?

is a function that takes one input.

Differentiation incomplete

*differentiation-with-respect-to-x*

or for brevity ,is a function with its input y.

Differentiation complete

*the result of taking the derivative-with-respect-to-x of y*

[1] Power Rule

[2] Product Rule

[3] Quotient Rule

[4] Chain Rule.

Use this when you have a composition function (in the form )

First differentiate f(x). Then differentiate g(x)

Chain Rule – Exponential

*Common derivative*

Chain Rule – Log

*Common derivative*

Chain Rule – Root

Chain Rule – Chain Rule with the Product Rule

*Product rule -> Chain Rule*

*Chain rule -> Product Rule*

)

[5] Implicit Differentiation

*find the derivative of y with respect to x*

*without having to solve the given equation for y.*

*product rule*

*isolate*

**Lesson 1**

Integration

This is the antiderivative of a function. It is used for calculating things like areas, volumes, and central points.

integral

derivative of integral

function

what does all this mean?

integral of the function

infinitesimal displacement along x

limits of integration

[1] Power Rule

[2] Trig

[3] U Substitution

Study List

[ ] Sign patterns

[ ] Optimization

[ ] Rolle’s Theorem

[ ] Mean Value Theorem

[ ] Area between curves, solids of revolution

[x] Integration

[x] Integration U Substitution

[ ] Integration by parts

[ ] Taylor polynomials

Revision List

[ ] Trig identities/Formulas

[ ] Derivatives – Extrema

[ ] Derivatives – Integrals

[ ] Derivatives – Common Integrals